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13. ABSTRACT (Maximum 200 words)

The objective of this project was to further develop the theory of controlled diffusion processes and its applications. The main line of research deals with control of dissipative stochastic systems which can be viewed as a model of an aircraft under uncertain wind conditions or perturbation in the mechanical unit.

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Principal Investigator Michael Taksar

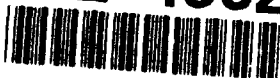
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This work is a continuation of the research conducted under the auspices of AFOSR grant 87-0278.

The objective of this project was to further develop the theory of controlled diffusion processes and its applications. The main line of research deals with control of dissipative stochastic systems which can be viewed as a model of an aircraft under uncertain wind conditions or perturbation in the mechanical unit.

The principal investigator visited the French National Institute in Information and Automatic Control (INRIA) doing joint research with French scientists on multidimensional singular control theory. This visit was supported by NATO international exchange grant awarded in conjunction with the AFOSR grant. In the previous years the theory of singular stochastic control was being developed mainly for the stationary or infinite horizon case. There, the time-parameter does not enter explicitly the corresponding Hamilton-Jacobi-Bellman equation. Non-stationary analysis for the singular control was done so far only for the one-dimensional case. It appears that the recent

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work of the principal investigator at INRIA will enable to make progress in the finite-horizon multidimensional singular control theory.

Another avenue of research deals with general linear additive controls and general convex control cost structure. In such problems the number of control functionals  $m$  is not necessarily the same as the dimension  $n$  of the process, and each coordinate of the process is controlled not by one functional rather by a linear combination of all  $m$  functionals. The cost of instantaneous displacement in this model is proportional to the length of displacement with coefficient of proportionality dependent on the direction. The proportionality coefficients in turn are determined by a norm in the  $m$ -dimensional space (the so called Minkowski norm). The unit ball in this norm can be any convex set containing zero. Control problems with such cost structure arise in inventory theory and network of queues, when one uses diffusion approximations for the underlying stochastic processes. Menaldi and Taksar are currently completing the second phase of research on singular control with general convex cost structure. This is a large problem which has been only partially solved.

There is an important issue in stochastic control problem, namely: - why one usually minimizes the expected cost? While there are other criteria, this one seems to be the most useful and the easiest to deal with. It turns out that for an ergodic problem, the one which deals with expected average cost, this criterion is much stronger than one could anticipate. Namely, the policy which is optimal in the mean in the ergodic problem is also optimal in probability and almost surely. The latter means that any other policy

yields larger average cost almost surely. This phenomenon is shown in the paper by Presman, Rotar, and Taksar.

Significant progress was achieved in studying diffusion approximation for controlled stochastic systems. In a series of papers by the principal investigator and collaborators different controlled stochastic systems (queueing, inventory, manufacturing) in heavy traffic were considered and their limiting behavior was studied. It was shown that in each case there exists an appropriate singular control problem solvable in closed form which provides an asymptotically optimal policy for the original system. These results bring both justification of diffusion approximation for controlled queueing/inventory models as well as a practical way of dealing with such models when they cannot be solved explicitly.

Two graduate students were supported by the grant. Paulo Santana was supported in his graduate studies by this grant at Florida State University. He has successfully completed his research on the problems associated with singular control on a finite time interval. He has completed the writing of his thesis "Finite Horizon Singular Control and a Related Two-Person Game". He defended his thesis in Spring of 1988. In his work he has rigorously proved the link between the optimal singular control on a finite time interval and an optimal stopping game of two persons with opposite interests. Charu Srivastava was supported in her graduate studies at the State University of New York at Stony Brook. She is currently finishing her thesis "Optimal Policies in Queueing and Manufacturing Models" which she expects to defend in Spring 1992.

The results of the research conducted under the auspices of AFOSR were presented at the Eighth and Ninth International Conference on Analysis and Optimization of Systems, Antibes, France, June, 1988, and June 1990 and at the 27th IEEE International Conference on Decision and Control, Austin, Texas, 1988, and at the International Conference on Control and application in Jerusalem, Israel, April, 1989, and at the joint TIMS-SOBRAPO International Conference, Rio de Janeiro, Brazil, 1991. Below is a list of work conducted under the auspices of the AFOSR grant 88-0183.

1. KANTOROVICH'S FUNCTIONALS IN THE SPACE OF MEASURES by S.T. Rachev and M. Taksar (submitted to Stochastics).

The paper studies the problem of finding measures on Cartesian product with given marginals in such a way that a specified integral functional attains the minimal value. It analyzes relationships between different functionals and associated distances between the marginal distributions. The latter problem relates to the classical Monge-Kantorovich problem of translocation of masses, to the problem of the best allocation policy, the problem of optimal training of recruits for the set of jobs, etc.

2. OPTIMAL CORRECTION PROBLEM OF A MULTIDIMENSIONAL STOCHASTIC SYSTEM by J.L. Menaldi and M. Taksar Automatica. 25 (1989), 223-232.

We consider a dissipative system under uncertainty governed by a multidimensional diffusion process. We are allowed to correct the position of the system by a means of additive functional. The cost of correction is proportional to the length of the displacement of the position of the system induced by the functional with coefficients dependent on the direction of displacement. The objective is to minimize the expected cumulative cost associated with the position of the system and the amount of control exerted. It is shown that Hamilton-Jacobi-Bellman's equation of the problem has a solution which corresponds to the optimal cost of the problem. An existence of optimal policy is proved.

3. AN ASYMPTOTIC ANALYSIS OF HIERARCHICAL CONTROL OF MANUFACTURING SYSTEMS BY H.M. Soner, J. Lehoczky, S. Sethi and M. Taksar, Mathematics of Operations Research, 16 (1991), 3, 596-608.

This paper presents an asymptotic analysis of a hierarchical manufacturing system with machines subject to breakdown and repair. The machine fluctuations are much faster than the accumulation and discounting of costs, and this gives rise to a limiting problem in which the stochastic machine availability is replaced by the equilibrium mean availability. The value function for the original problem converges to the value function of the limiting problem. The limiting problem is computationally tractable and sometimes has a closed form solution.

4. EXACT BOUNDS FOR THE DIFFERENCE BETWEEN A MEAN AND A MEDIAN by M. Taksar (technical report).

We show that if a distribution has the second moment then the maximal distance between its mean and its median does not exceed its standard deviation  $\sigma$ . If we restrict ourselves only to the distributions with unimodal densities then the same difference does not exceed  $\sigma/1.6$ . In both cases the bounds are sharp. The latter problem is reduced to the problem of finding minimal variance in the class of all nonnegative random variables with monotone densities and with means equal to 1.

5. SINGULAR CONTROL FOR MULTIDIMENSIONAL GAUSSIAN PROCESSES by J.L. Menaldi, M. Robin and M. Taksar, Mathematics of Controls, Signals and Systems, 5 (1992), 93-116.

This work extends the results of the singular control theory in two directions. Firstly, it considers the underlying process with nonconstant drift coefficients. Secondly, it considers ergodic optimality criteria, rather than usual discounted criteria.

6. EXPLICIT SOLUTION OF A GENERAL CONSUMPTION/PORTFOLIO PROBLEM WITH SUBSISTENCE CONSUMPTION AND BANKRUPTCY, by S. Sethi, M. Taksar and E. Presman, to appear in Journal of Economic Dynamics and Control.

This paper gives a closed form solution for a general continuous-time single agent consumption and portfolio decision problem with subsistence consumption. The analysis allows for general continuously differentiable concave utility functions. The model takes into consideration that consumption must be no smaller than a given subsistence rate and that bankruptcy can occur.

7. INFINITE HORIZON INVESTMENT CONSUMPTION MODEL WITH NONTERMINAL BANKRUPTCY, by S. Sethi and M. Taksar, Journal of Optimization Theory and Applications, 74 (1992).

This paper solves a general continuous-time consumption and portfolio decision problem for a single agent for whom there exists upon bankruptcy, a possibility of recovery from his bankruptcy. The main contribution of the paper is in the modeling of the recovery process. Moreover, it is shown that the model with recovery has a one-to-one correspondence with the model with terminal bankruptcy treated in the literature.

8. SKOROHOD PROBLEMS WITH NONSMOOTH BOUNDARY CONDITIONS, by M. Taksar, to appear in Journal of Computational and Applied Mathematics.

A stochastic differential equation in a region with oblique reflection on the boundary is considered. The existence of a weak solution to the Skorohod problem in the region is proved without requirement on the smoothness of the boundary or the reflection vector-field.

9. OPTIMALITY IN PROBABILITY AND ALMOST SURELY FOR AN INFINITE HORIZON LINEAR REGULATOR PROBLEM by E. Presman, V. Rotar and M. Taksar, to appear in Stochastics.

We consider a classical linear regulator stochastic control model in which the objective is to minimize the expected limiting average (ergodic in another terminology) cost. We show that the optimal stationary Markov policy  $u^*$  for this problem is also optimal in a much stronger sense. Namely, it minimizes the limiting average cost almost surely, thus being uniformly better than any other policy.

We also compare  $u^*$  with any family of policies  $u_T$ . It is shown that probability that the average cost on  $[0, T]$  associated with  $u^*$  exceeds the one associated with  $u_T$  by  $\epsilon$ , converges to zero as  $T \rightarrow \infty$ .

10. DIFFUSION APPROXIMATION FOR GI/G/1 CONTROLLED QUEUES by E. Krichagina, and M. Taksar, submitted to Queueing Systems.

A queueing model is considered in which there is a cost associated with the length of the queue. The cost is represented by a holding cost function  $h$ . A controller can increase the service rate incurring the service cost proportional to the increased rate with coefficient  $l$ . The objective is to minimize the total expected discounted cost.

When  $h$  and  $l$  are small the control problem can be approximated by a singular stochastic control problem for Brownian motion, namely, the so-called "reflected follower" problem. The optimal policy in this problem is characterized by a single number  $z$  so that the optimal process is a reflected diffusion in  $[0, z]$ .

11. A REGENERATIVE GENERALIZED ELASTIC SCREEN PROCESS AS A HEAVY TRAFFIC LIMIT FOR QUEUES WITH OPTIMAL INTERRUPTIONS by O. Kella and M. I. Taksar, submitted to Mathematics and Operations Research.

We consider a queue in which the server is available for primary customers for a random number of busy cycles, after which he leaves for a random amount of time as soon as the system becomes empty. Under appropriate normalization we establish a heavy traffic limit which turns out to be a regenerative generalized elastic screen process (RGESP) with random jumps or linear parts of the trajectories. These jumps or linear parts occur when the local time at zero accumulates to a certain random level. We compare the stationary distribution for the RGESP with the corresponding one for the M/G/1 queue, where for both explicit expressions are obtained, and show that the results are consistent.

12. PRODUCTION CONTROL IN A FAILURE-PRONE MANUFACTURING SYSTEM: DIFFUSION APPROXIMATION AND ASYMPTOTIC OPTIMALITY by E. Krichagina, S. Lou, S. Sethi and M. I. Taksar, to appear in Annals of Applied Probability.

We consider the problem of controlling the production rate of a single machine, single product, stochastic manufacturing system in order to minimize the total discounted inventory/backlog costs. The demand has two components: one deterministic with a constant rate  $d$  and the other stochastic with random demand batches. Under the heavy loading (or heavy traffic) condition, i.e., when the average production capacity is close to the average demand, control problem can be solved explicitly. The solution is then interpreted in terms of the actual manufacturing system and a control policy for this system is derived. We prove that the resulting policy is nearly optimal under the heavy traffic condition.

13. DOUBLE BAND POLICY FOR STOCHASTIC MANUFACTURING SYSTEMS IN HEAVY TRAFFIC by S. Lou, E. Krichagina and M. I. Taksar (technical report).

We consider the control of a manufacturing system producing one product. The demand for this product is random. The manager can produce the product at a fixed rate  $r$ , or to stop the production. Each time the production is resumed, a setup cost must be paid. To prevent the backlog, which is not allowed, the manager can buy the product from outside vendors, paying a fixed order cost and a variable cost. There is also a linear inventory holding cost. The objective is to minimize the total expected discounted cost. Under heavy traffic condition, i.e., when the production capacity is close to the average demand, this problem is approximated by an impulse control problem for Brownian motion, which is solved explicitly.



14. ASYMPTOTICALLY OPTIMAL POLICIES FOR CONTROLLED QUEUES IN HEAVY TRAFFIC by E. Krichagina and M. I. Taksar (technical report).

We consider GI/G/1 queueing system operating in heavy traffic. There is a proportional holding cost with coefficient  $h$ . A controller can increase the service rate (thus moving the system into light traffic) incurring the cost proportional to the increased rate with coefficient  $l$ . Given the discount rate  $\rho$  the objective is to minimize the total expected discounted cost. We show that when the parameters  $\delta$  associated with heavy traffic and  $\delta=h/l$  are small and  $\rho$  has an appropriate order this control problem can be approximated by a singular control problem for Brownian motion, namely, reflected follower problem.

Research in Progress

1. MULTIDIMENSIONAL SINGULAR CONTROL PROBLEMS WITH GENERAL CONTROL COSTS by J.L. Menaldi and M. Taksar.

This work deals with multidimensional problems in which a number of control functionals is different from the dimension of the underlying stochastic process. Each coordinate of the stochastic process is controlled by a linear combination of the functionals. This is the situation which arises in diffusion models of network of queues.

2. SINGULAR CONTROL IN VIDALE-WOLFE ADVERTISING MODEL by M. Taksar and S. Zhu.

A singular stochastic control is used to treat a classical Vidale-Wolfe advertising model to find an optimal level of production and advertisement of a large firm in a market subject to saturation.